

## AMENDMENTS TO THE CLAIMS

This listing of claims will replace all prior versions, and listings, of claims in the application:

### Listing of Claims:

1           1.     (Currently amended) A method for using interval techniques  
2     within a computer system to solve a multi-objective optimization problem,  
3     comprising:  
4           receiving a representation of multiple objective functions ( $f_1, \dots, f_n$ ) at  
5     the computer system, wherein ( $f_1, \dots, f_n$ ) are scalar functions of a vector  
6      $\mathbf{x} = (x_1, \dots, x_n)$ ;  
7           receiving a representation of a domain of interest for the multiple  
8     objective functions;  
9           storing the representations in a memory within the computer system; and  
10          performing an interval optimization process to compute guaranteed  
11     bounds on a Pareto front for the objective functions ( $f_1, \dots, f_n$ ), wherein for  
12     each point on the Pareto front, an improvement in one objective function cannot  
13     be made without adversely affecting at least one other objective function;  
14          wherein performing the interval optimization process involves applying a  
15     direct-comparison technique between subdomains of the domain of interest to  
16     eliminate subdomains that are certainly dominated by other subdomains,  
17          wherein performing the interval optimization process involves applying a  
18     gradient technique to eliminate subdomains that do not contain a local Pareto  
19     optimum.

wherein a subdomain  $[x]_i$  is eliminated by the gradient technique if an intersection of certainly negative gradient regions  $C_j$  for each objective function  $f_j$  is non-empty,  $\bigcap_{j=1}^n C_j([x]_i) \neq \emptyset$ , and

wherein the certainly negative gradient region  $C_j$  for objective function  $f_j$  is the intersection of  $\underline{N}_j([x]_i)$  (the negative gradient region associated with the minimum angle  $\underline{\theta}_j$  of the gradient of  $f_j$  over the subdomain  $[x]_i$ ) and  $\overline{N}_j([x]_i)$  (the negative gradient region associated with the maximum angle  $\overline{\theta}_j$  of the gradient of  $f_j$  over the subdomain  $[x]_i$ ).

2. (Cancelled)

3. (Cancelled)

4. (Currently amended) The method of ~~claim 2~~ claim 1, wherein the method further comprises iteratively:

bisecting remaining subdomains that have not been eliminated by the gradient technique; and

applying the gradient technique to eliminate bisected subdomains that do not contain a local Pareto optimum.

5. (Original) The method of claim 4, wherein bisecting a subdomain involves bisecting the subdomain in the direction that has the largest width of partial derivatives of all objective functions  $(f_1, \dots, f_n)$  over the subdomain.

6. (Original) The method of claim 4, wherein the direct-comparison technique is applied once for every  $n$  iterations of the gradient technique.

1           7.       (Original) The method of claim 6, wherein the iterations continue  
2 until either a predetermined maximum number of iterations are performed, or the  
3 largest area of any subdomain is below a predetermined value.

1           8.       (Original) The method of claim 1,  
2 wherein a subdomain  $U$  certainly dominates a subdomain  $V$  if every point  
3  $u \in U$  dominates every point  $v \in V$ ; and  
4 wherein a point  $u$  dominates a point  $v$  under minimization if,  
5  $u_i \neq v_i, i = 1, \dots, n$ , and  
6  $u_i < v_i$  for some  $i \in \{1, \dots, n\}$ .

1           9.       (Currently amended) A computer-readable storage medium storing  
2 instructions that when executed by a computer cause the computer to perform a  
3 method for using interval techniques within a computer system to solve a multi-  
4 objective optimization problem, wherein the computer-readable storage medium  
5 can be any device that can store code and/or data for use by a computer system,  
6 the method comprising:  
7 receiving a representation of multiple objective functions  $(f_1, \dots, f_n)$  at  
8 the computer system, wherein  $(f_1, \dots, f_n)$  are scalar functions of a vector  
9  $\mathbf{x} = (x_1, \dots, x_n)$ ;  
10 receiving a representation of a domain of interest for the multiple  
11 objective functions;  
12 storing the representations in a memory within the computer system; and  
13 performing an interval optimization process to compute guaranteed  
14 bounds on a Pareto front for the objective functions  $(f_1, \dots, f_n)$ , wherein for  
15 each point on the Pareto front, an improvement in one objective function cannot  
16 be made without adversely affecting at least one other objective function;

wherein performing the interval optimization process involves applying a  
 direct-comparison technique between subdomains of the domain of interest to  
 eliminate subdomains that are certainly dominated by other subdomains,  
 wherein performing the interval optimization process involves applying a  
 gradient technique to eliminate subdomains that do not contain a local Pareto  
 optimum,  
 wherein a subdomain  $[x]_i$  is eliminated by the gradient technique if an  
 intersection of certainly negative gradient regions  $C_j$  for each objective function  $f_j$   
 is non-empty,  $\bigcap_{j=1}^n C_j([x]_i) \neq \emptyset$ , and  
 wherein the certainly negative gradient region  $C_j$  for objective function  $f_j$   
 is the intersection of  $\underline{N}_j([x]_i)$  (the negative gradient region associated with the  
 minimum angle  $\underline{\theta}_j$  of the gradient of  $f_j$  over the subdomain  $[x]_i$ ) and  $\overline{N}_j([x]_i)$  (the  
 negative gradient region associated with the maximum angle  $\overline{\theta}_j$  of the gradient of  
 $f_j$  over the subdomain  $[x]_i$ ).

10. (Cancelled)

11. (Cancelled)

12. (Currently amended) The computer-readable storage medium of  
 claim 11, wherein the method further comprises iteratively:  
 bisecting remaining subdomains that have not been eliminated by the  
 gradient technique; and  
 applying the gradient technique to eliminate bisected subdomains that do  
 not contain a local Pareto optimum.

1           13.     (Original) The computer-readable storage medium of claim 12,  
2     wherein bisecting a subdomain involves bisecting the subdomain in the direction  
3     that has the largest width of partial derivatives of all objective functions ( $f_1, \dots,$   
4      $f_n$ ) over the subdomain.

1           14.     (Original) The computer-readable storage medium of claim 12,  
2     wherein the direct-comparison technique is applied once for every  $n$  iterations of  
3     the gradient technique.

1           15.     (Original) The computer-readable storage medium of claim 14,  
2     wherein the iterations continue until either a predetermined maximum number of  
3     iterations are performed, or the largest area of any subdomain is below a  
4     predetermined value.

1           16.     (Original) The computer-readable storage medium of claim 9,  
2     wherein a subdomain  $U$  certainly dominates a subdomain  $V$  if every point  
3      $u \in U$  dominates every point  $v \in V$ ; and  
4     wherein a point  $u$  dominates a point  $v$  under minimization if,  
5                  $u_i \neq v_i, i = 1, \dots, n$ , and  
6                  $u_i < v_i$  for some  $i \in \{1, \dots, n\}$ .

1           17.     (Currently amended) An apparatus that uses interval techniques to  
2     solve a multi-objective optimization problem, comprising:  
3         a receiving mechanism configured to receive a representation of multiple  
4     objective functions ( $f_1, \dots, f_n$ ), wherein ( $f_1, \dots, f_n$ ) are scalar functions of a  
5     vector  $\mathbf{x} = (x_1, \dots, x_n)$ ;  
6         wherein the receiving mechanism is configured to receive a representation  
7     of a domain of interest for the multiple objective functions;

8 a memory configured to store the representations; and  
 9 an interval optimizer configured to performing an interval optimization  
 10 process to compute guaranteed bounds on a Pareto front for the objective  
 11 functions  $(f_1, \dots, f_n)$ , wherein for each point on the Pareto front, an  
 12 improvement in one objective function cannot be made without adversely  
 13 affecting at least one other objective function;  
 14 wherein the interval optimizer is configured to apply a direct-comparison  
 15 technique between subdomains of the domain of interest to eliminate subdomains  
 16 that are certainly dominated by other subdomains,  
 17 wherein the interval optimizer is configured to apply a gradient technique  
 18 to eliminate subdomains that do not contain a local Pareto optimum,  
 19 wherein a subdomain  $[x]_i$  is eliminated by the gradient technique if an  
 20 intersection of certainly negative gradient regions  $C_j$  for each objective function  $f_j$   
 21 is non-empty,  $\bigcap_{j=1}^n C_j([x]_i) \neq \emptyset$ , and  
 22 wherein the certainly negative gradient region  $C_j$  for objective function  $f_j$   
 23 is the intersection of  $N_j([x]_i)$  (the negative gradient region associated with the  
 24 minimum angle  $\underline{\theta}_j$  of the gradient of  $f_j$  over the subdomain  $[x]_i$ ) and  $\overline{N}_j([x]_i)$  (the  
 25 negative gradient region associated with the maximum angle  $\overline{\theta}_j$  of the gradient of  
 26  $f_j$  over the subdomain  $[x]_i$ ).

1 18. (Cancelled)

1 19. (Cancelled)

1 20. (Currently amended) The apparatus of ~~claim 19~~claim 17, wherein  
 2 the interval optimizer is configured to iteratively:

3           bisect remaining subdomains that have not been eliminated by the gradient  
4   technique; and to  
5           apply the gradient technique to eliminate bisected subdomains that do not  
6   contain a local Pareto optimum.

1           21.    (Original) The apparatus of claim 20, wherein bisecting a  
2   subdomain involves bisecting the subdomain in the direction that has the largest  
3   width of partial derivatives of all objective functions ( $f_1, \dots, f_n$ ) over the  
4   subdomain.

1           22.    (Original) The apparatus of claim 20, wherein the direct-  
2   comparison technique is applied once for every  $n$  iterations of the gradient  
3   technique.

1           23.    (Original) The apparatus of claim 22, wherein the iterations  
2   continue until either a predetermined maximum number of iterations are  
3   performed, or the largest area of any subdomain is below a predetermined value.

1           24.    (Original) The apparatus of claim 17,  
2           wherein a subdomain  $U$  certainly dominates a subdomain  $V$  if every point  
3    $\mathbf{u} \in U$  dominates every point  $\mathbf{v} \in V$ ; and  
4           wherein a point  $\mathbf{u}$  dominates a point  $\mathbf{v}$  under minimization if,  
5                        $u_i \neq v_i, i = 1, \dots, n$ , and  
6            $u_i < v_i$  for some  $i \in \{1, \dots, n\}$ .